Numerical Computing

Lecture 10: Numerical Integration

Francisco Richter Mendoza

Università della Svizzera Italiana
Faculty of Informatics, Lugano, Switzerland

Lecture Overview

- **Quadrature Basics**: Nodes, weights, degree of precision
- **Newton–Cotes**: Midpoint, Trapezoid, Simpson; composite rules
- **Gaussian Quadrature**: Gauss–Legendre optimality
- **Adaptive Methods**: Adaptive Simpson; Romberg overview
- **Multi-D Integration**: Tensor products; Monte Carlo

Numerical Quadrature

Problem

Approximate $I[f] = \int_a^b f(x)\,dx$ by $Q[f] = \sum_{i=0}^n w_i f(x_i)$.

- **Degree of precision**: highest degree integrated exactly
- **Peano kernel**: general error representation

Newton–Cotes Nodes

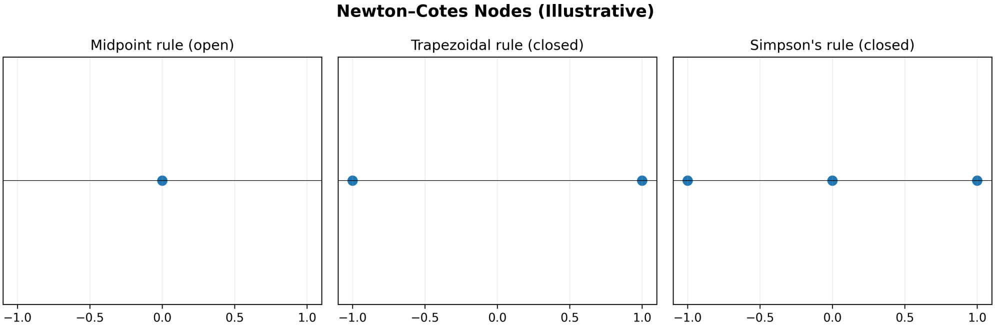

Figure: Node layouts for midpoint (open), trapezoid (closed), and Simpson's rules.

Convergence: Trapezoid vs Simpson

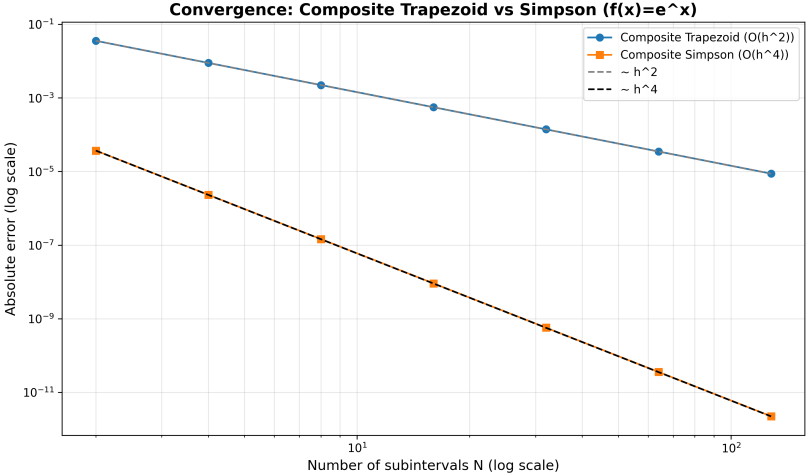

Figure: Composite trapezoid ($O(h^2)$) vs composite Simpson ($O(h^4)$) on $f(x) = e^x$.

Gaussian Quadrature vs Composite Simpson

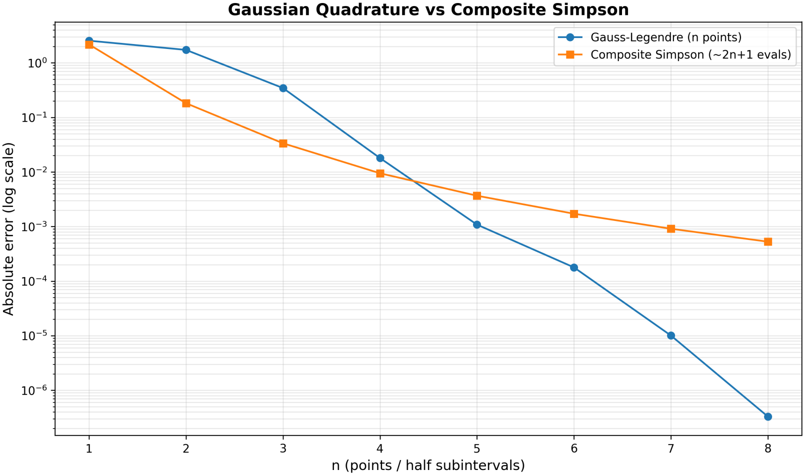

Figure: Gauss–Legendre achieves far higher accuracy for smooth integrands.

Adaptive Simpson (Illustrative)

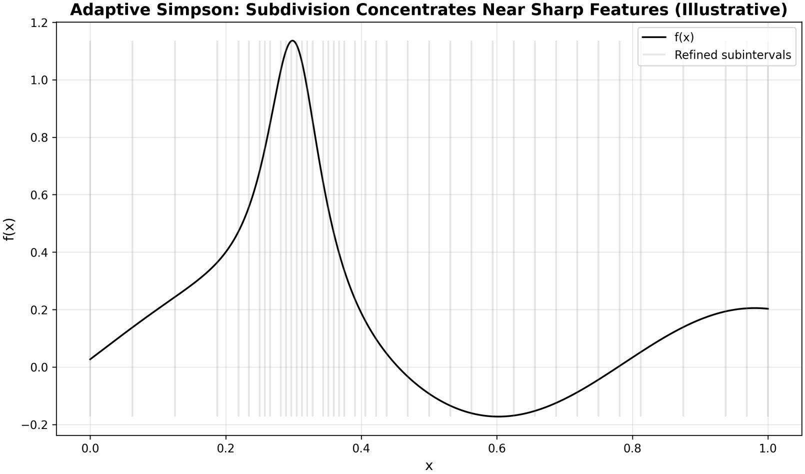

Figure: Subdivision concentrates where features are sharp.

Integration Methods Overview

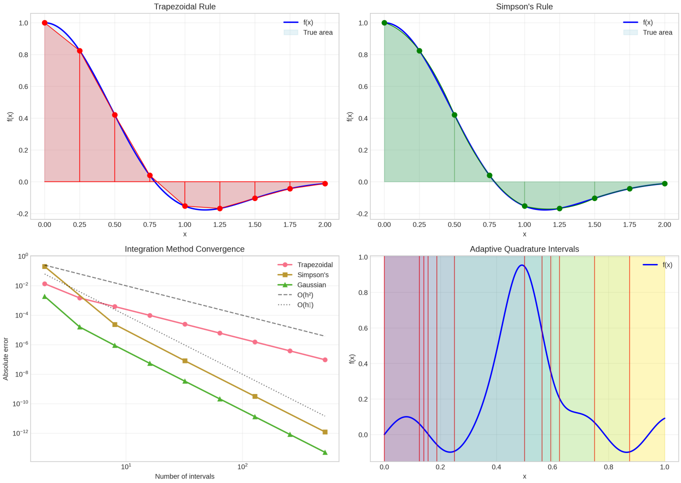

Figure: Geometric intuition and convergence behavior of quadrature families.

Key Takeaways

- **Order**: higher order reduces error faster for smooth f
- **Optimality**: Gaussian quadrature attains degree $2n - 1$
- **Adaptivity**: adjust effort where the integrand is complex
- **Practical**: composite Simpson is a strong baseline